HEAT TRANSFER IN A REGION OF ACCELERATED FLOW WHEN A SYSTEM OF TWO-DIMENSIONAL STREAMS IS INCIDENT ON A FLAT SURFACE NORMAL TO IT

P. N. Romanenko and M. I. Davidzon

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The heat transfer problem is considered when a flat plate is heated by a system of two-dimensional streams directed at right angles to the surface.

Two-dimensional streams which are incident at right angles to a surface (Fig. 1) find wide application for the intensification of heat and mass transfer. But our knowledge of this subject is very limited [1-3].

The present investigation was made with an experimental apparatus, the working part of which is in the form of a convergent chamber ending in three pipes. The pipes were made of steel plate of thickness 6 mm. To reduce the perturbing effect of the inner surfaces of the pipes on the characteristics of the emergent flow, the surfaces were machined on a grinding machine. The edges of the pipe at their outlets were carefully fitted to each other so that in the fully closed state the gap was less than 0.1 mm along the whole length of the pipe. The length of the pipes corresponded to the width of the lower part of the chamber and was 250 mm. The height of the pipes was 100 mm. The width of the gap in the pipes was $H_0 = 5$ mm. The central pipe was securely fixed. Screw mechanisms were used to position the other two pipes at the required distance from the central pipe. In accordance with the dimensions of the chamber and the design of the pipes, the pitch S of the pipes could be varied within the limits 85-205 mm. Thus, it was possible to construct a system of three plane-parallel streams (Fig. 1).

The design of the apparatus provided for the possibility of changing the distance h from the end of the pipes to the plate within the limits h = 0-250 mm.

Air reached the working part of the apparatus from an open, continuously operational wind tunnel. Before the air entered the settling chamber it passed through a grating with two rows to eliminate large scale vortex formation.

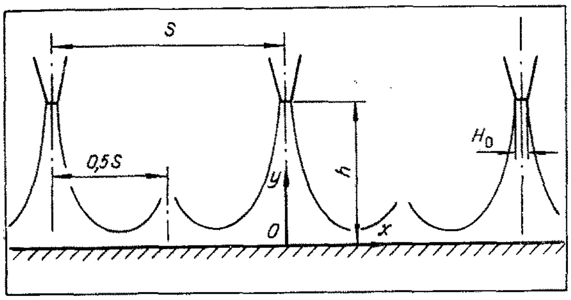

Fig. 1. Diagram of the flow of a system of two-dimensional streams onto a flat surface.

In studying the hydrodynamics of the flow of streams onto a flat surface, a highly polished brass plate of dimensions 1000 × 240 × 5 mm was used. The plate had 23 holes in it of diameter 0.5 mm to sample the pressure.

During the experiments the dynamic air pressure in the initial part of the stream was measured with a Prandtl tube of intake diameter 0.5 mm. The total pressure at cross sections of the stream above the plate was measured with a Pitot tube of intake dimensions 0.4 × 0.8 mm. A microcoordinator was used to move the Pitot tube. The accuracy of the displacements was 0.05 mm in the vertical direction and 0.1 mm in the horizontal

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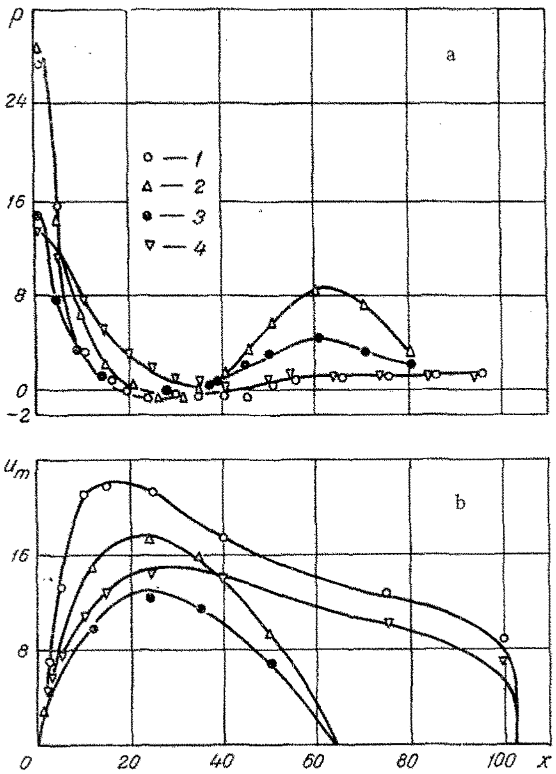

Fig. 2. The pressure distribution along the surface (a) and the velocity u_m at the outer boundary of the boundary layer (b): 1) $Re_0 = 9.03 \cdot 10^3$; $\bar{S} = 40.6$; $\bar{h} = 10$; 2) $9.09 \cdot 10^3$; 25; 10; 3) $6.73 \cdot 10^3$; 25; 10; 4) $9.09 \cdot 10^3$; 40.6; 20 ($p \cdot 10^4$, bar; x, mm; u_m, m/sec).

direction. The initial point for measurements was fixed with respect to an electrocontact.

The measuring instruments were micromanometers with an inclined tube of type MMN. The accuracy class was 1.

In calculating the velocities the pressure at the plate was taken as the static pressure.

In experiments to study the local heat transfer coefficients the hot air streams interacted with a plate composed of separate calorimeters. Each heat element (calorimeter) was in the form of a tube of rectangular cross section. The working surface of the calorimeter was of copper plate of thickness 3 mm soldered to the milled steel body. The heat was removed with running water. The working surface of the heat element was 240 × 10 mm. The calorimeters, isolated from each other by laminated Bakelite packing of thickness 1 mm, were assembled into the form of a plate for subsequent polishing. The working surface of the plate was machined to mirror brightness.

The transfer coefficients were determined by the stationary thermal regime method for t_w = const. The time needed to obtain the stationary thermal regime was 3.5-4 h. In the experiments not only the hydrodynamical parameters, but also the air temperature at the outlet from the pipes and at cross sections above the plate, the temperature of the working surface of each heat element, the temperature difference of the water at the inlet and outlet of the calorimeter, the temperature drop at the lower wall of the calorimeter, and the volume of water passing through each calorimeter were measured. The temperature sensors were chrome–constantan thermocouples $\phi = 0.2$ mm. The temperature measuring instrument was a control potentiometer KP-59. The heat transfer coefficients were calculated for each heat element separately from the heat balance equation. Heat losses by radiation were computed from the standard relations. Convective heat losses from lower wall of the calorimeter were determined from the temperature drop at the wall, measured by a differential (triple) hyperthermocouple. The heat transfer coefficients, computed at the surface of one calorimeter, were taken as the local values α.

The defining parameters had the following ranges in the experiments:

$$H_0 = 5\ \text{mm};\ Re_0 = (3 - 12.3)\cdot 10^3;\ \bar{h} = 10 - 28;\ \bar{S} = 25 - 40.6;\ \bar{T}_w = T_0/T_w = 1.16 \pm 0.03;\ T_0 \approx 350°\ \text{K}.$$

The experimental results were generalized with respect to the temperature of the flow at the end of the pipes.

The experimental results show that when the system of streams is incident on a surface normal to it, a complex dynamic flow pattern is created. Until it reaches the plate, each stream of the system develops as a free stream.* After it impacts on the plate the stream is turned through 90 deg and propagates along the surface.

In the impact region the kinetic energy of the free stream is transformed into potential energy and potential energy is transformed into the kinetic energy of the stream bounded by the solid surface.

Another feature of the interaction between the system of incident streams is that neighboring streams meet as they spread out over the surface of the solid body. This results in part of the kinetic energy being transformed into pressure energy. As a result there is motion in the boundary layer with a positive longitudinal pressure gradient.

*We studied only systems of two-dimensional streams for which there was no interaction between the separate streams in a freely expanding segment.

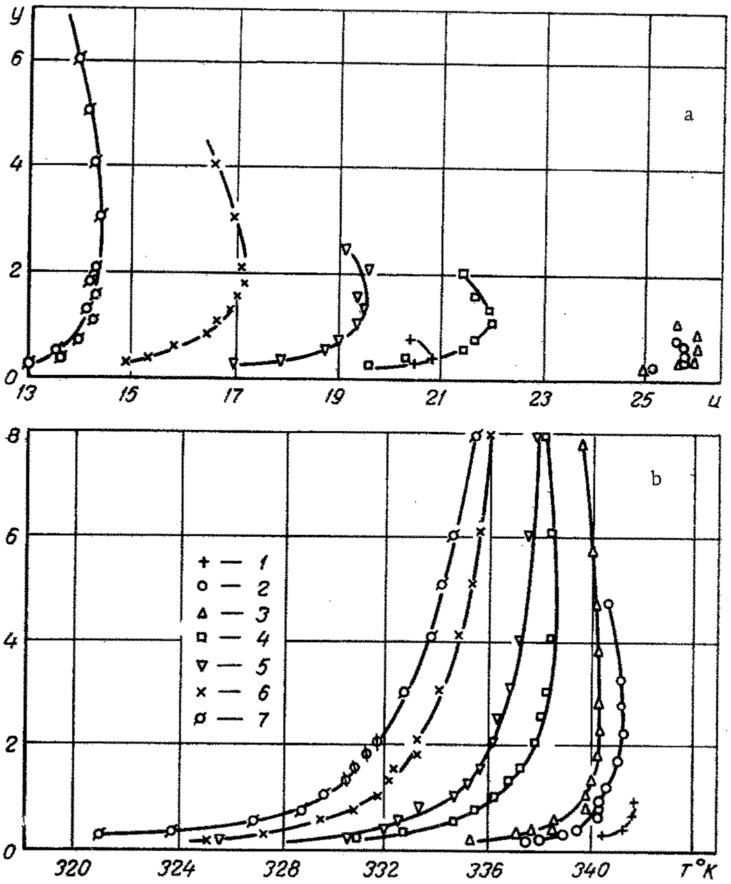

Fig. 3. Velocity profile (a) and temperature profile (b) at cross sections of the stream above the plate for experimental conditions ($Re_0 = 12.3 \cdot 10^3$; $\overline{S} = 40.6$; $\overline{h} = 20$; $\overline{T}_0 = 348.6$°K): 1) x = 12; x/S = 0.059; 2) 23; 0.113; 3) 34; 0.168; 4) 49; 0.241; 5) 70; 0.345; 6) 81; 0.399; 7) 93; 0.457 (y, mm; u, m/sec; T, °K).

The pressure distribution curves at the plate (Fig. 2a) indicate the presence of longitudinal pressure gradients. Independently of the parameters of the system, the pressure diminishes from a maximum at the critical point to some minimal value (above or below atmospheric) and then increases again.

The pressure change, like other flow characteristics, is periodic in nature with period x = 0.5S.

Hence the dynamic and thermal interactions of the system of two-dimensional streams were studied in a segment of length x = 0.5S.

Figure 3 shows the velocity profile (Fig. 3a) and the temperature profile (Fig. 3b) for one of the temperatures. The velocity changes at each section of the stream, as in the semi-bounded streams, from u = 0 at the wall to some maximum $u = u_m$ at $y = \delta$ and then falls to the velocity of the surrounding medium.

The change in the velocity at the outer edge of the boundary layer along the surface is shown in Fig. 2b. The velocity u_m changes from zero at the critical point to a maximum for given parameters of the system of $u_m = u_*$ at $x = x_*$ and then it falls to zero.

Hence, there are two flow regions – an accelerated region with dp/dx < 0 ($0 \le x \le x_*$, $0 \le u_m \le u_*$) and a retarded region with dp/dx > 0. The generalized experimental results show that in the region of accelerated flow

$$u_m = 1.58\, u_0\, (\overline{h})^{-0.5}\, (\overline{S})^{0.3} \overline{x}_*\, (1 - 0.5\overline{x}_*), \tag{1}$$

where $\overline{x}_* = x/x_*$.

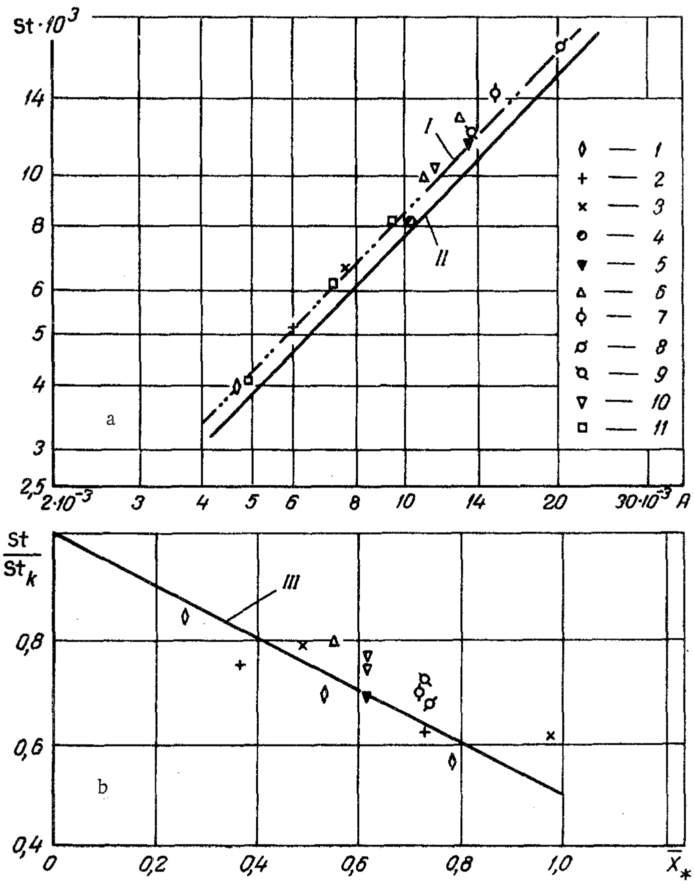

Fig. 4. Comparison of experimental and theoretical results for heat transfer in the neighborhood of the critical point (a), and in the region of accelerated flow (b): 1) $\bar{S} = 40.6$; $Re_0 = 12.2 \cdot 10^3$; $\bar{h} = 28$; 2) 40.6; $12.3 \cdot 10^3$; 20; 3) 40.6; $12.1 \cdot 10^3$; 15; 4) 40.6; $12.2 \cdot 10^3$; 10; 5) 30; $5.43 \cdot 10^3$; 10; 6) 25; $7.28 \cdot 10^3$; 10; 7) 40.6; $5.5 \cdot 10^3$; 10; 8) 40.6; $3.2 \cdot 10^3$; 10; 9) 40.6; $7.13 \cdot 10^3$; 10; 10) 30; $7.52 \cdot 10^3$; 10; 11) by [1]; I) by equation (19); II) by (15); III) by (14). $A = Re_0^{-0.5} (\bar{h})^{-0.75} (\bar{S})^{0.43}$.

The extent of the region of accelerated flow x_* is given by the equation

$$x_* = 2.48\, h\, (\bar{S})^{-0.57}. \tag{2}$$

Hydrodynamic investigations showed that in the neighborhood of the critical point there is a laminar boundary layer when a system of two-dimensional streams is incident normally on a surface, as in the case of a single axisymmetric stream [4-7]. As the distance from the critical point increases (in the region of accelerated flow) a transition from laminar to turbulent flow in the boundary layer is observed.

The local and average heat transfer coefficients in the region of accelerated flow can be computed using one of the approximate methods for calculating the laminar boundary layer and the empirical relations (1) and (2).

Following the Karman–Pohlhausen method in its modern form [8] the impulse equation for a two-dimensional incompressible boundary layer can be put in the following form for laminar flow:

$$u_m \frac{d}{dx}\left(\frac{\vartheta^2}{\nu}\right) = 0.47 - 6\left(\frac{\vartheta^2}{\nu}\right)\frac{du_m}{dx}. \tag{3}$$

Equation (3) has the solution

$$\frac{\vartheta^2}{\nu} = \frac{0.47}{u_m^6} \int u_m^5 dx + c. \tag{4}$$

Noting (1) and (2), and the initial conditions [8] for x = 0,

$$\varkappa = \frac{\vartheta^2}{\nu} du_m/dx = 0.077$$

equation (4) can be written

$$\frac{\vartheta^2}{\nu} = 0.123\, h u_0^{-1} (\bar{h})^{0.5} (\bar{S})^{-0.87} (1 - 0.5\,\bar{x}_*)^{-6} F(\bar{x}_*), \tag{5}$$

where

$$F(\bar{x}_*) = 1 - 2.15\bar{x}_* + 1.875\bar{x}_*^2 - 0.833\bar{x}_*^3 + 0.1875\bar{x}_*^4 - 0.017\bar{x}_*^5.$$

At the limits of the region of accelerated flow $F(\bar{x}_*)$ has the approximation

$$F(\bar{x}_*) = (1 - 0.5\bar{x}_*)^4. \tag{6}$$

The ratio of the thickness of the impulse losses to the thickness of the hydrodynamical boundary layer can be computed approximately by linearizing the relation between the first $\Lambda = \delta^2/\nu\ du_m/dx$ and the second $\varkappa = \theta^2/\nu\ du_m/dx$ form parameters. At the boundaries ($\varkappa_k = 0.77$ and $\varkappa_* = 0$) the approximation must give the exact value of the ratio of these parameters. Under these conditions, $\vartheta/\delta = 0.1048$, and, noting (5) and (6),

$$\delta = 3.34\ \frac{\nu^{0.5}\, h^{0.5}\, (\bar{h})^{0.25}}{u_0^{0.5}\, (\bar{S})^{0.43} (1 - 0.5\bar{x}_*)}\ . \tag{7}$$

From the equation for the heat flow density

$$q_w = -\lambda \left(\frac{\partial t}{\partial y}\right)_{y=0} = \alpha\,(t_m - t_w)$$

we define the heat transfer coefficient

$$\alpha = -\lambda \left(\frac{\partial t}{\partial y}\right)_{y=0} \Big/ (t_m - t_w). \tag{8}$$

Let us assume that the temperature distribution in the boundary layer is

$$t = a + by + cy^2 + dy^3 + ey^4 + ny^5. \tag{9}$$

For the boundary conditions

$$\begin{gathered} y = 0 \quad t = t_w = \text{const},\ \partial^2 t/\partial y^2 = 0, \\ y = \delta_\text{T} \quad t = t_m,\ \frac{\partial t}{\partial y} = 0,\ \frac{\partial^2 t}{\partial y^2} = 0,\ \frac{\partial^3 t}{\partial y^3} = 0 \end{gathered} \tag{10}$$

the temperature gradient at the wall is determined by the equation

$$\left(\frac{\partial t}{\partial y}\right)_{y=0} = 2.5\ \frac{t_m - t_w}{\delta_\text{T}}. \tag{11}$$

By (8) and (11) the heat transfer coefficient is

$$\alpha = 2.5\ \lambda/\delta_\text{T}. \tag{12}$$

The relation between the thickness of the thermal and dynamic boundary layers [8] is given by

$$\delta_\text{T}/\delta = \text{Pr}^{-1/3}. \tag{13}$$

By substituting (13) and (7) in (12) we can obtain a computational expression for the local values of the nondimensional heat transfer coefficient in the region of accelerated flow

$$\text{St} = 0.75\,\text{Re}_0^{-0.5}\text{Pr}^{-2/3}\,(\bar{h})^{-0.75}\,(\bar{S})^{0.43}\,(1 - 0.5\bar{x}_*). \tag{14}$$

In the neighborhood of the critical point, $\bar{x}_* \to 0$ and

$$\text{St}_k = 0.75\,\text{Re}_0^{-0.5}\text{Pr}^{-2/3}\,(\bar{h})^{-0.75}\,(\bar{S})^{0.43}. \tag{15}$$

An equation for calculating the heat transfer in the neighborhood of the critical point can also be obtained from the exact solution of the equation of motion in the neighborhood of the critical point. Such a solution was obtained by Khiments and Howarth [8]. From these calculations the thickness of the dynamical boundary layer of a two-dimensional flow in the neighborhood of the critical point is

$$\delta = 2.4\sqrt{\nu/L}, \tag{16}$$

where L is known from experiment or theory.

In solving the problem, Khiments and later Howarth assumed that $u_m = Lx$. Under the conditions of the problem we studied

$$L = \left|\frac{du_m}{dx}\right|_{x\to 0}. \tag{17}$$

After differentiating (1) and using (17), we find L and therefore δ:

$$\delta = 3\sqrt{\frac{\nu(\bar{h})^{0.5}h}{u_0(\bar{S})^{0.87}}}. \tag{18}$$

Then, following the familiar procedure for calculating St from (12), taking (13) and (18) into account, we have

$$\mathrm{St}_k = 0.832\,\mathrm{Re}_0^{-0.5}\mathrm{Pr}^{-2/3}(\bar{h})^{-0.75}(\bar{S})^{0.43}. \tag{19}$$

The local heat transfer coefficients, calculated from (19) are 11.2% higher than those determined from (15).

Figure 4 shows a comparison between the experimental results and equations (15) and (19) for heat transfer in an air stream (Pr = 0.72) in the neighborhood of the critical point (Fig. 4a) and the change in the local heat transfer coefficients in the region of accelerated flow (Fig. 4b).

We see that the experimental results agree satisfactorily with the solutions we have obtained.

NOTATION

ϑ is the thickness of the impulse losses;
u_m is the velocity at the outer edge of the boundary layer;
H_0 is the calibre of the pipe (the smaller dimension of the gap in the pipe);
x is the current distance from the critical point;
$\bar{x}_* = x/x_*$;
x_* is the extent of the region of accelerated flow (for $x = x_*$, $u_m = u_*$);
h is the distance from the end of the pipe to the plate;
$\bar{h} = h/H_0$;
S is the pitch of the pipes;
$\bar{S} = S/H_0$;
t_w is the temperature of the surface of the plate;
t_m is the temperature at the outer edge of the wall boundary;
t_0 is the air temperature at the end of the pipe;
u_0 is the air velocity at the exit from the pipe;
$\mathrm{Re}_0 = (u_0H_0)/\nu$;
$\mathrm{St} = \alpha/c_p\rho u_0$.

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